General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Thursday 15 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A curve passes through the point $(0, 1)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{1 + x^2}$$

Starting at the point $(0, 1)$, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 0.4$. Give your answer to five decimal places. *(5 marks)*

2 The complex number $2 + 3\mathrm{i}$ is a root of the quadratic equation

$$x^2 + bx + c = 0$$

where b and c are real numbers.

(a) Write down the other root of this equation. *(1 mark)*

(b) Find the values of b and c. *(4 marks)*

3 Find the general solution of the equation

$$\tan\left(\frac{\pi}{2} - 3x\right) = \sqrt{3}$$

(5 marks)

4 It is given that

$$S_n = \sum_{r=1}^{n} (3r^2 - 3r + 1)$$

(a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = n^3$. *(5 marks)*

(b) Hence show that $\sum_{r=n+1}^{2n} (3r^2 - 3r + 1) = kn^3$ for some integer k. *(2 marks)*

5 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k:

(i) $\mathbf{A} + \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{A}^2$. *(2 marks)*

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. *(4 marks)*

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) The matrix **A** represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(3 marks)*

6 A curve has equation

$$y = \frac{(x-1)(x-3)}{x(x-2)}$$

(a) (i) Write down the equations of the three asymptotes of this curve. *(3 marks)*

(ii) State the coordinates of the points at which the curve intersects the x-axis. *(1 mark)*

(iii) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{(x-1)(x-3)}{x(x-2)} < 0$$ *(2 marks)*

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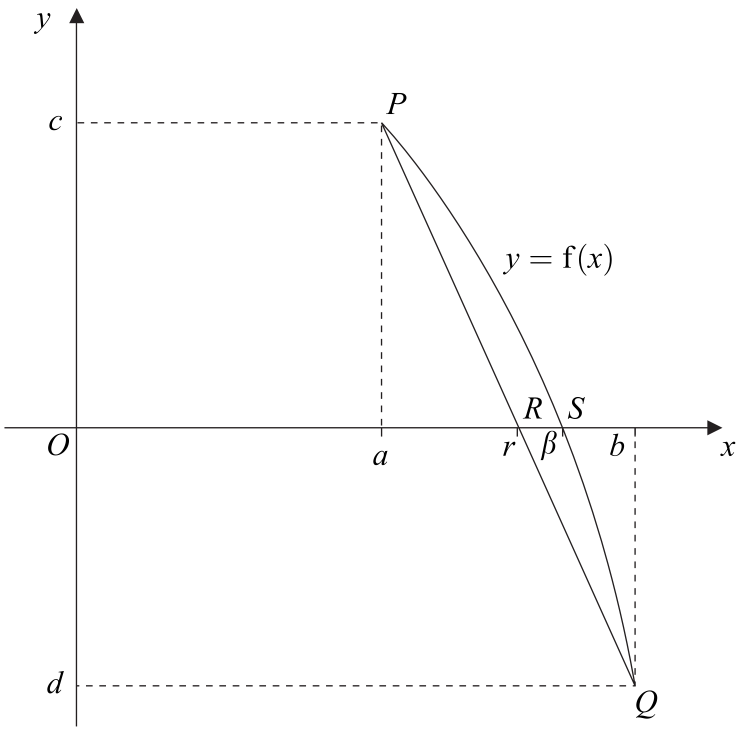
7 The points $P(a, c)$ and $Q(b, d)$ lie on the curve with equation $y = \mathrm{f}(x)$. The straight line PQ intersects the x-axis at the point $R(r, 0)$. The curve $y = \mathrm{f}(x)$ intersects the x-axis at the point $S(\beta, 0)$.



(a) Show that

$$r = a + c\left(\frac{b-a}{c-d}\right)$$ *(4 marks)*

(b) Given that

$$a = 2,\ b = 3 \text{ and } \mathrm{f}(x) = 20x - x^4$$

(i) find the value of r; *(3 marks)*

(ii) show that $\beta - r \approx 0.18$. *(3 marks)*

8 For each of the following improper integrals, find the value of the integral **or** explain why it does not have a value:

(a) $\displaystyle\int_1^\infty x^{-\frac{3}{4}}\,\mathrm{d}x\,;$ *(3 marks)*

(b) $\displaystyle\int_1^\infty x^{-\frac{5}{4}}\,\mathrm{d}x\,;$ *(3 marks)*

(c) $\displaystyle\int_1^\infty \left(x^{-\frac{3}{4}} - x^{-\frac{5}{4}}\right)\mathrm{d}x\,.$ *(1 mark)*

9 A hyperbola H has equation

$$x^2 - \frac{y^2}{2} = 1$$

(a) Find the equations of the two asymptotes of H, giving each answer in the form $y = mx$. *(2 marks)*

(b) Draw a sketch of the two asymptotes of H, using roughly equal scales on the two coordinate axes. Using the same axes, sketch the hyperbola H. *(3 marks)*

(c) (i) Show that, if the line $y = x + c$ intersects H, the x-coordinates of the points of intersection must satisfy the equation

$$x^2 - 2cx - (c^2 + 2) = 0$$ *(4 marks)*

(ii) Hence show that the line $y = x + c$ intersects H in two distinct points, whatever the value of c. *(2 marks)*

(iii) Find, in terms of c, the y-coordinates of these two points. *(3 marks)*

END OF QUESTIONS

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